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10EC046

M.Tech. Degree Examination, June/July 2011
Linear Algebra

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \end{bmatrix}.$$ **(08 Marks)**

b. If A, B, C are matrices over the field F such that the products BC and A(BC) are defined, then so are the products AB, (AB)C then prove that A(BC) = (AB)C. **(06 Marks)**

c. Solve the following system of equations :

$x_1 - 2x_2 + x_3 = 0$
$2x_2 - 8x_3 = 8$
$-4x_1 + 5x_2 + 9x_3 = -9.$ **(06 Marks)**

2 a. Solve the system of linear equations using LU factorization method with $U_{i1} = 1$

$x_1 + 2x_2 + 3x_3 = 0$
$2x_1 + 2x_2 + 3x_3 = 3$
$-x_1 - 3x_2 = 2.$ **(10 Marks)**

b. If W_1 and W_2 are finite dimensional subspaces of a vector space V_1 then $W_1 + W_2$ is finite dimensional and $\dim W_1 + \dim W_2 = \dim (W_1 \cap W_2) + \dim (W_1 + W_2)$. **(08 Marks)**

c. Define a linearly dependent set and basis of vector space V. **(02 Marks)**

3 a. Let V be finite dimensional vector space over the field F and let $\{\alpha_1 \ldots \alpha_n\}$ be an ordered basis for V. Let W be a vector space over the same field F and let $\{\beta_1 \ldots \beta_n\}$ be any vectors in W. Then show that there is precisely one linear transformation T from V in to W such that $T_{\alpha j} = \beta_j, j = 1, 2, \ldots n$. **(08 Marks)**

b. Given a matrix $A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 1 & 0 \end{bmatrix}$. Determine the linear transformation $T : V_3(R) \rightarrow V_2(R)$ relative to the basis β_1 and β_2 given by $\beta_1 = \{(1, 1, 1)\,(1, 2, 3)\,(1, 0, 0)\}$, $\beta_2 = \{(1, 1)\,(1, -1)\}$. **(05 Marks)**

c. Let $T : V \rightarrow W$ be a linear transformation, then
i) R(T) is subspace of W
ii) N(T) is a subspace V
iii) T is one to one iff $N(T) = \{0\}$. **(07 Marks)**

4 a. If T is a linear transformation from V into W where V and W are vector spaces over the field F and V is finite dimensional then prove that rank (T) + nullity (T) = dim V. **(10 Marks)**

b. Find the range, null space, rank and nullity of linear transformation $T : V \rightarrow W$ defined by $T(x, y, z) = (y - x, y - z)$. Also verify rank nullity theorem. **(05 Marks)**

c. Let V and W be vector spaces over the field. Let T and U be linear transformations from V into W. Show that the function T + U defined by $(T + U)\,\alpha = T\alpha + U\alpha$ is a linear transformation. **(05 Marks)**

5 a. Let T be a linear operator on the finite dimensional space V. Let $C_1 \ldots C_k$ be the distinct characteristic values of T and let W_i be the space of characteristic vectors associated with the characteristic value C_i. If $W = W_1 + \ldots + W_k$, then prove that $\dim W = \dim W_1 + \ldots + \dim W_k$. Also show that if B_i is an ordered basis for W_i, then $B = (B_1 \ldots B_k)$ is an ordered basis for W. **(07 Marks)**

b. Let T be a linear operator on an n-dimensional vector space V(or let A be an n × n matrix). Show that the characteristic and minimal polynomials for T have the same roots, except for multiplication. **(07 Marks)**

c. Let W be an invariant subspace for T. Then prove the following :

i) The characteristic polynomial for the restriction operator T_w divides the characteristic polynomial for T

ii) The minimal polyn9omial for T_w divides the minimal polynomial for T. **(06 Marks)**

6 a. Construct an orthogonal basis for W given

$$x_1 = \begin{bmatrix}1\\1\\1\\1\end{bmatrix}, x_2 = \begin{bmatrix}0\\1\\1\\1\end{bmatrix}, x_3 = \begin{bmatrix}0\\0\\1\\1\end{bmatrix}.$$ **(10 Marks)**

b. Find the QR factorization of

$$A = \begin{bmatrix}1 & -1 & -1\\1 & 0 & 0\\1 & -1 & 0\\0 & 1 & 1\end{bmatrix}.$$ **(10 Marks)**

7 a. Find a least –square solution for $A_x = b$ for

$$A = \begin{bmatrix}1 & 1 & 0 & 0\\1 & 1 & 0 & 0\\1 & 0 & 1 & 0\\1 & 0 & 1 & 0\\1 & 0 & 0 & 1\\1 & 0 & 0 & 1\end{bmatrix}, b = \begin{bmatrix}-3\\-1\\0\\2\\5\\1\end{bmatrix}.$$ **(12 Marks)**

b. For any linear operator T on a finite dimensional inner product space V, show that there exists a unique linear operator T^* on V such that $(T\alpha | \beta) = (\alpha | T^* \beta)$ for all $\alpha, \beta \in V$. **(08 Marks)**

8 a. Convert the quadratic form $Q(x) = x_1^2 - 8x_1x_2 - 5x_2^2$ into quadratic form with no cross products. **(07 Marks)**

b. Let $A = \begin{bmatrix}3 & 2 & 1\\2 & 3 & 1\\1 & 1 & 4\end{bmatrix}$. Find the maximum value of the quadratic form x^TAx subject to the constraint $x^Tx = 1$ and find a unit vector at which this maximum value is attained. **(07 Marks)**

c. Find the maximum and minimum values of $Q(x) = 9x_1^2 + 4x_2^2 + 3x_3^2$ subject to $x^Tx = 1$. **(06 Marks)**

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